6.5 Use Proportionality Theorems

Before You used proportions with similar triangles.

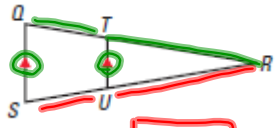
Now You will use proportions with a triangle or parallel lines.

Why? So you can use perspective drawings, as in Ex. 28.

THEOREMS *For Your Notebook*

THEOREM 6.4 Triangle Proportionality Theorem

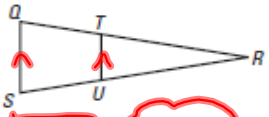
If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

THEOREM 6.5 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.



If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

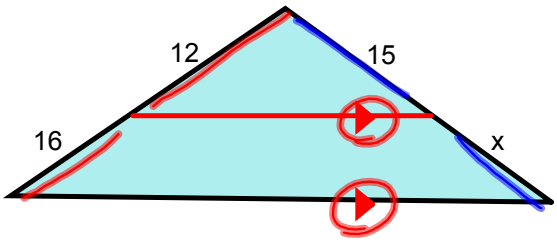
Example 1:

Use the ***Triangle Proportionality Theorem*** to find *x* in the triangle.

$\frac{12}{16} = \frac{15}{x}$

$\frac{12x}{12} = \frac{240}{12}$

$x = 20$



Example 2:

Use the ***Triangle Proportionality Theorem*** to find *x* in the triangle.

$\frac{3}{4} = \frac{2}{x-2}$

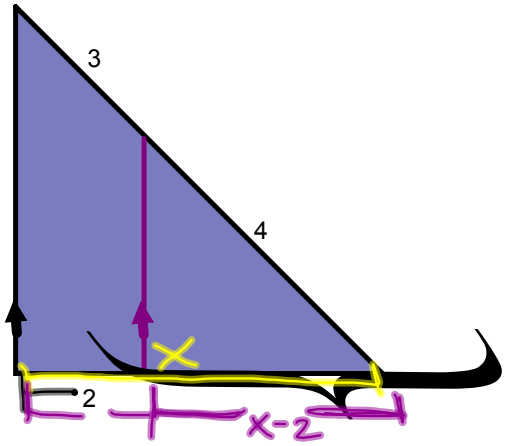
$3(x-2) = 8$

$3x - 6 = 8$

$+6 \quad +6$

$\frac{3x}{3} = \frac{14}{3}$

$x = 14/3$



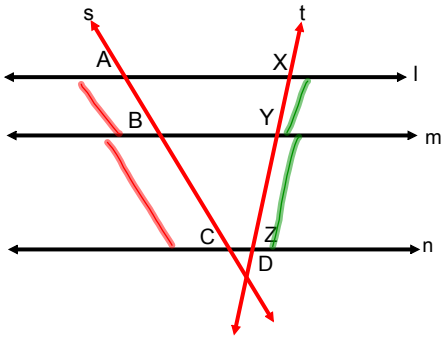
$x = 14/3$ or $4\frac{2}{3}$

Two-Transversal Proportionality Corollary

Three or more parallel lines divide two intersecting transversals proportionally.

IOW: ~~$\frac{AB}{BC} = \frac{XY}{YZ}$~~

$$\frac{AB}{BC} = \frac{XY}{YZ}$$

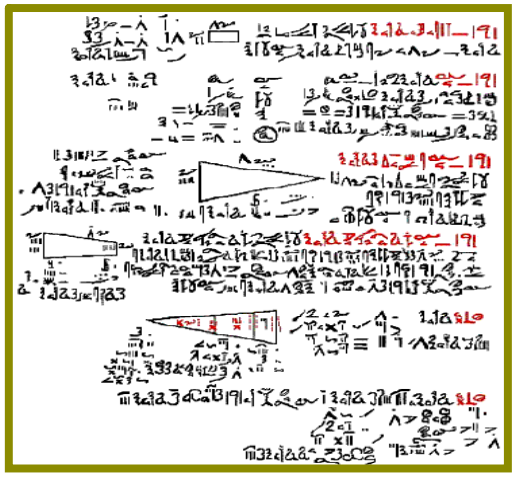


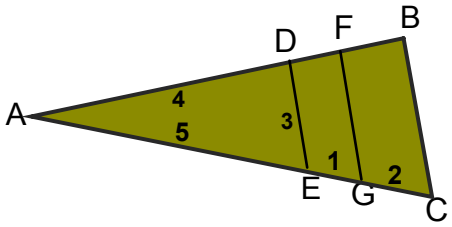
In class:

Example 3:

Students in ancient Egypt studied geometry to solve practical problems involving the pyramids. This problem is based on a problem in a papyrus copied in 1650 B.C.E. by the scribe Ahmes from a source that may date back to 2000 B.C.E.

Find DF, FB, FG and BC.

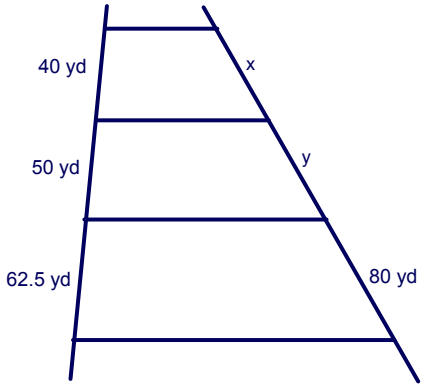




In class:

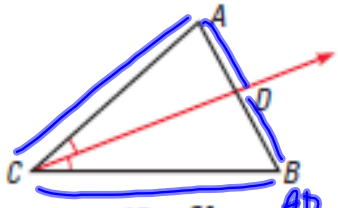
Example 4:

Land plots between two streets are laid out according to the plan shown below. The horizontal lot boundaries are parallel to each other. Find the missing lengths.



THEOREM 6.7

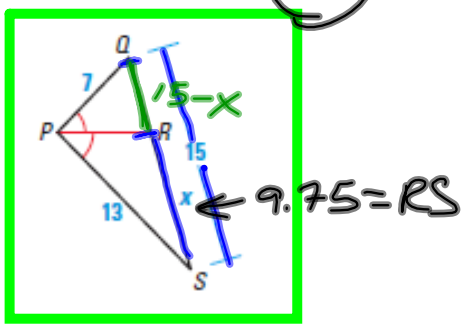
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



$\frac{AD}{DB} = \frac{CA}{CB}$ or $\frac{AD}{CA} = \frac{DB}{CB}$

Example 5:

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of $\overline{RS}$.



$\frac{15-x}{7} = \frac{x}{13}$

$13(15-x) = 7x$

$195 - 13x = 7x$

$+13x \quad +13x$

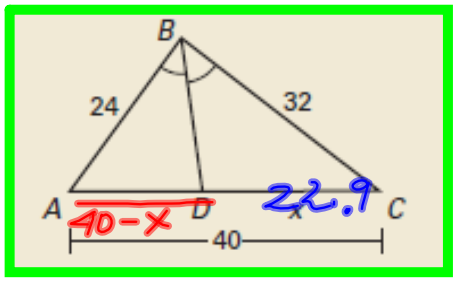
$195 = 20x$

$\frac{195}{20} = \frac{20x}{20}$

$x = 9.75$

Example 6:

In the diagram, $\angle ABD \cong \angle CBD$. Use the given side lengths to find the length of $\overline{DC}$.



$\frac{24}{40-x} = \frac{32}{x}$

$24x = 32(40-x)$

$24x = 1280 - 32x$

$+32x \quad +32x$

$\frac{56x}{56} = \frac{1280}{56}$

$x = 22\frac{6}{7} \approx 22.9$

Questionnaire!